AD-A193 697 POLYNOMIAL DEFINITION OF DISCRETE FIELD POINTS OF MAP 1/1
OF DIFFUSION EQUATION PART 1(U) ARMY BALLISTIC RESEARCH
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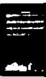

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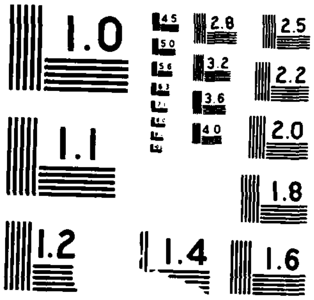
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The diffusion equation is solved by differencing methods to present discrete values of the dependent variable at specific times and locations. The solution is given as coordinate polynomials and is bounded between 0 and ∞ .

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

INTRODUCTION

The diffusion equation of Physics has been used to analyze unsteady heat transfer, boundary layer velocity distribution, long line electrical voltage fluctuation, and salt-solute penetration. The general mathematical expression is

$$\partial T/\partial t = a\ \partial^2 T/\partial \chi^2 ,$$

where particular physical constraints determine the context. There are two approaches to the problem statement and solution; the one most widely used being transformation, and the other, finite difference techniques, employ variations of summing averages of term values established by unique methods. This report considers an averaging type solution in algebraic format. The final result consists of a series of discrete polynomials with rational coefficients which describe the dependent variable at each time-distance coordinate in the manner of the non-reflecting Schmidt plot.

PROCEDURE

Essentially, the differential equation is recast as a finite difference expression which is transposed first to geometric and then to polynomial algebraic form. The polynomials, representing discrete solutions to the differential equation, are analyzed by differencing techniques whereby the numerical coefficients of common diagonal terms are found to be expressible in a generalized matrix.

From the one dimensional partial differential equation

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial \chi^2} , \qquad (1)$$

where T is the dependent variable, t is an independent variable, χ is an independent variable, and a is a constant. In heat transfer problems, T is temperature, t is time, χ is distance, and a is the diffusivity. Application of the finite difference procedure gives

$$\frac{1}{a} \frac{\Delta_t T}{\Delta t} = \frac{\Delta_\chi^2 T}{\Delta \chi^2} \qquad (2)$$

with Δt the finite difference in time, $\Delta \chi$ the finite difference in distance, $\Delta_t T$ the time variable effecting a change in T, and $\Delta_\chi T$ the distance variable effecting a change in T.
By expansion the equation becomes

$$\frac{1}{a} \left(\frac{T_n^{(t+1)} - T_n^{(t)}}{\Delta \tau} \right) = \frac{T_{(n+1)}^{(t)} - 2\,T_n^{(t)} + T_{(n-1)}^{(t)}}{\Delta \chi^2} , \qquad (3)$$

where subscript n refers to the χ increments and superscript t refers to the t increments. Schmidt[1] developed the graphical form shown on Figure 1 with the stepwise linear temperature gradients across adjacent layers of material. Since the change in internal energy within a layer of material over a finite time is the difference between the heat flow in and heat flow out, the corresponding temperature increment becomes a function of the ratio $\frac{\Delta\chi^2}{2a\Delta t}$ and it is convenient to select this ratio as unity, leading to

$$\Delta t = \frac{\Delta\chi^2}{2a} \quad . \tag{4}$$

Also, a geometric simplification results from defining the graphical proportions as

$$m = \frac{\Delta\chi}{\Delta\chi + \Delta\chi_o} \quad , \tag{5}$$

where the $\Delta\chi_o$ is the graphical analogue of the initial film coefficient.

Table 1 shows the discrete algebraic expressions for the time-temperature-distance intersections of Figure 1. Along the diagonals of Table 1 a matched power polynomial appears and the coordinate expression for T in time and distance takes the general form

$$T\,(N,P) = \frac{\varphi}{2^h \quad 2^j} \left(A - Bm + Cm^2 - \cdots \pm m^k\right) \tag{6}$$

where N is a distance index, P is a time index,

$$h = \frac{(P + N) - 2 - \left|\sin\,(P + N)\,\pi/2\right|}{2}$$

(the external denominator exponent), k is the terminal exponent of m, j is k minus the exponent of m (the individual term denominator exponent), and

$$\varphi = m\,(T_0^{\,0} - T_1^{\,0})$$

with A, B, C ... numerical coefficients of the interior terms of the equation. To establish T (N, P) for any φ and m (which include the physical constraints) it is necessary to establish the precise values of the coefficients A, B, C ...; and this is the object of the current investigation.

Any full expression, T (N,P), can be developed from a Gregory-Newton formulation[2] of the separate terms A, B, C, for the bounded diffusion equation as shown in Figure 2. Some interesting progressions do result but an alternate and more geometric presentation is available from the direct finite difference tables.

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[1] Sutton, G.P., Rocket Propulsion Elements, John Wiley & Sons, New York 1956.

[2] Spiegel, M.R., Theory and Problems of Finite Differences and Finite Difference Equations, Schaum's Outline Series in Mathematics, McGraw-Hill Book Company, New York, etc., 1971, pp. 36-44.

Table 2 is also extracted, by difference equation procedure, from Table 1 and generates the coefficients of the constant term, A, for the coordinate expression of distance and time. The starting point is within the heavy box of column 7. These numbers, 17548, 25147, 35401, 49024 and 66868, were found by direct calculation using Figure 1 and Table 1 and are the constant numerator terms only. Table 3 lists the complete polynomial expressions for these coordinates. By the usual differencing, the 7th through the 1st columns are established. It is then possible to work vertically using the regression in column 2, back to zero; and then to complete the elements of columns 1 through 6. Noting the resulting bias progression at the tops of these columns, the next step is continue diagonally (I, II, III) to column 8 and, using column 7 as a summation, verify the vertical sequence of column 8. Columns 9, 10, 11, ..., are generated similarly.

Within the individual frames containing the coefficients is a parenthesized pair of numbers which indicate the distance and time coordinate. These indices run diagonally upwards at constant distance and bi-sequentially as time. Tables 4 through 9 are formed by the same procedure and extend arbitrarily to the 6th power of m. However, a different sequence appears along diagonals I, II and III according to the power of m. Table 10 summarizes this behavior and reveals yet another correlation, shown mainly by column 4, from which the adjacent columns can be constructed adinfinitum. The final coincidence occurs from a reinspection of Tables 2, and 4-9 where the digital vertical counting column (1, 2, 3, 4, 5 ...) conjoins the power of m and the time sequence of the first distance diagonal (1,3), (1,5), (1,7), etc., by an interval of 3 in the counter according to Table 11.

RESULTS

To write any term defining the dependent variable in time and distance, Tables 2 through 11 are used to form the numerical coefficients in Equation (6),

$$T\,(N,P) = \frac{\phi}{2^{h}2^{j}}\left[A - Bm + Cm^{2} - \ldots + \ldots \pm m^{K}\right]$$

For T (1,15) for instance,

$$N = 1,$$

$$P = 15,$$

$$k = \frac{P - N - |\sin (P - N)\,\pi/2|}{2} = \frac{(15 - 1) - |\sin 7\pi|}{2} = 7,$$

$$h = \frac{P + N - 2\,|\sin (P + N)\,\pi/2|}{2} = \frac{(15 + 1) - 2 - |\sin 8\pi|}{2} = 7, \text{ and}$$

$$j = (k - \text{term exponent of } m) = (7 - \text{term exponent of } m)\ .$$

The finite difference tables for the (1,15), j variables are then reconstructed (Tables 2, and 4 -9) using Tables 10 and 11 and the respective numerators determined, yielding

whereby

Term	Numerator Value	Exponent of "m"	"j"
A	51480	0	7
B	39796	1	6
C	20264	2	5
D	7050	3	4
E	1672	4	3
F	260	5	2
G	24	6	1
H	1	7	0

hence

$$T\,(1,15) = \frac{\phi}{128}\left[\frac{51480}{128} - \frac{39796m}{64} + \frac{20264m^2}{32} - \frac{7050m^3}{16} + \frac{1672m^4}{8} - \frac{260m^5}{4} + \frac{24m^6}{2} - m^7\right].$$

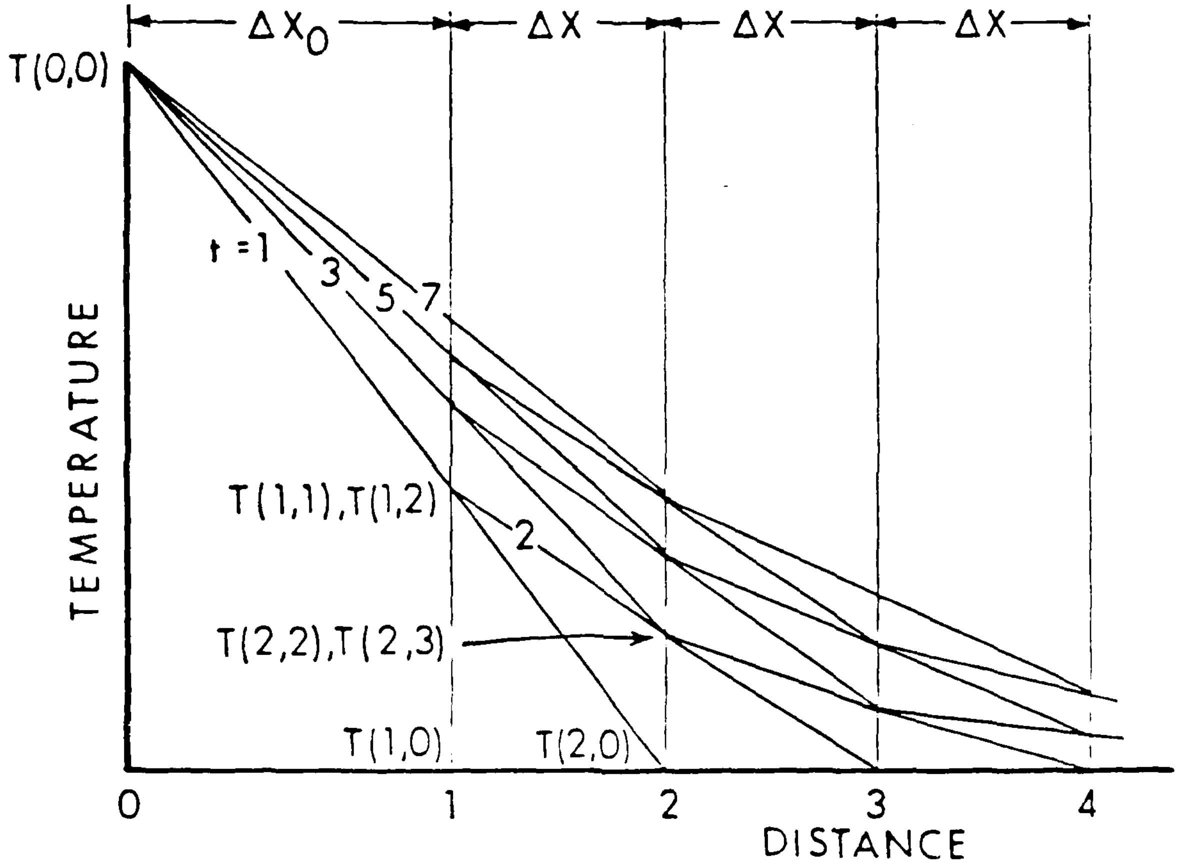


Figure 1. Schmidt Diagram

TABLE 1. TIME LOCATION

N	P →					
↓	1	2	3	4	5	6
1	$\varnothing$	$\varnothing$	$(\varnothing/2)(3 - m)$	$(\varnothing/2)(3 - m)$	$(\varnothing/4)(30/4 - 9/2\ m + m^2)$	$(\varnothing/4)(30/4 - 9/2\ m + m^2)$
2		$\varnothing/2$	$\varnothing/2$	$(\varnothing/4)(7/2 - m)$	$(\varnothing/4)(7/2 - m)$	$(\varnothing/8)(38/4 - 10/2\ m + m^2)$
3			$\varnothing/4$	$\varnothing/4$	$(\varnothing/8)(4 - m)$	$(\varnothing/8)(4 - m)$
4				$\varnothing/8$	$\varnothing/8$	$(\varnothing/16)(9/2 - m)$
5					$\varnothing/16$	$\varnothing/16$
6						$\varnothing/32$
7						
8						

TABLE 1. (continued)

7	8
$(\phi/8)(35/2 - 29/2\ m + 12/2\ m^2 - m^3)$	$(\phi/8)(35/2 - 29/2\ m + m^2 - m^3)$
$(\phi/8)(35/4 - 10/2\ m + m^2)$	$(\phi/16)(187/8 - 69/4\ m + 13/2\ m^2 - m^3)$
$(\phi/16)(47/4 - 11/2\ m + m^2)$	$(\phi/16)(47/4 - 11/2\ m + m^2)$
$(\phi/16)(9/2 - m)$	$(\phi/32)(57/4 - 12/2\ m + m^2)$
$(\phi/32)(5 - m)$	$(\phi/32)(5 - m)$
$\phi/32$	$(\phi/64)(11/2 - m)$
$\phi/64$	$\phi/64$
	$\phi/128$

TABLE 1. (continued)

9
$(\phi/16)(630/16 - 325/8\ m + 95/4\ m^2 - 15/2\ m^3 + m^4)$
$(\phi/16)(187/8 - 69/4\ m + 13/2\ m^2 - m^3)$
$(\phi/32)(244/8 - 81/4\ m + 14/2\ m^2 - m^3)$
$(\phi/32)(57/4 - 12/2\ m + m^2)$
$(\phi/64)(68/4 - 13/2\ m + m^2)$
$(\phi/64)(11/2 - m)$
$(\phi/128)(6 - m)$
$\phi/128$
$\phi/256$

TABLE 1. (continued)

10
$(\O/16)(630/16 - 325/8\ m + 95/4\ m^2 - 15/2\ m^3 + m^4)$
$(\O/32)(874/16 - 406/8\ m + 109/4\ m^2 - 10/2\ m^3 + m^4)$
$(\O/32)(244/8 - 81/4\ m + 14/2\ m^2 - m^3)$
$(\O/64)(312/8 - 94/4\ m + 15/2\ m^2 - m^3)$
$(\O/64)(68/4 - 13/2\ m + m^2)$
$(\O/128)(80/4 - 14/2\ m + m^2)$
$(\O/128)(6 - m)$
$(\O/256)(13/2 - m)$
$\O/256$
$\O/512$

TABLE 1. (continued)

11
$(\phi/32)(1386/16 - 843/8\ m + 312/4\ m^2 - 141/4\ m^3 + 18/2\ m^4 - m^5)$
$(\phi/32)(874/16 - 406/8\ m + 109/4\ m^2 - 16/2\ m^3 + m^4)$
$(\phi/64)(1186/16 - 500/8\ m + 124/4\ m^2 - 17/2\ m^3 + m^4)$
$(\phi/64)(312/8 - 94/4\ m + 15/2\ m^2 - m^3)$
$(\phi/128)(392/8 - 298/4\ m + 16/2\ m^2 - m^3)$
$(\phi/128)(60/4 - 14/2\ m + m^2)$
$(\phi/256)(93/4 - 15/2\ m + m^2)$
$(\phi/256)(13/2 - m)$
$(\phi/512)(14/2 - m)$
$\phi/512$
$\phi/1024$

TABLE 1. (continued)

12
$(\phi/32)(1386/16 - 843/8\ m + 312/4\ m^2 - 141/4\ m^3 + 18/2\ m^4 - m^5)$
$(\phi/64)(1979/16 - 1093/8\ m + 374/4\ m^2 - 79/2\ m^3 + 19/2\ m^4 - m^5)$
$(\phi/64)(1186/16 - 500/8\ m + 124/4\ m^2 - 17/2\ m^3 + m^4)$
$(\phi/128)(1578/16 - 608/8\ m + 140/4\ m^2 - 18/2\ m^3 + m^4)$
$(\phi/128)(392/8 - 298/4\ m + 16/2\ m^2 - m^3)$
$(\phi/256)(485/8 - 123/4\ m + 17/2\ m^2 - m^3)$
$(\phi/256)(93/4 - 15/2\ m + m^2)$
$(\phi/512)(214/8 - 16/2\ m + m^2)$
$(\phi/512)(14/2 - m)$
$(\phi/1024)(15/2 - m)$
$\phi/1024$
$\phi/2048$

TABLE 1. (continued)

13
$(\text{Ø}/64)(3003/16 - 4165/16\ m + 1841/8\ m^2 - 532/4\ m^3 - 196/4\ m^4 + 21/2\ m^5 - m^6)$
$(\text{Ø}/64)(1979/16 - 1093/8\ m + 374/4\ m^2 - 79/2\ m^3 + 19/2\ m^4 - m^5)$
$(\text{Ø}/128)(2762/16 - 1397/8\ m + 444/4\ m^2 - 88/2\ m^3 + 20/2\ m^4 - m^5)$
$(\text{Ø}/128)(1578/16 -608/8\ m + 140/4\ m^2 - 18/2\ m^3 + m^4)$
$(\text{Ø}/256)(2063/16 - 731/8\ m + 157/4\ m^2 - 19/2\ m^3 + m^4)$
$(\text{Ø}/256)(485/8 - 123/4\ m + 17/2\ m^2 - m^3)$
$(\text{Ø}/512)(592/8 - 139/4\ m + 17/2\ m^2 - m^3)$
$(\text{Ø}/512)(214/8 -16/2\ m + m^2)$
$(\text{Ø}/1024)(244/8 - 17/2\ m + m^2)$
$(\text{Ø}/1024)(15/2 - m)$
$(\text{Ø}/2048)(16/2 - m)$
$\text{Ø}/2048$
$\text{Ø}/4096$

TABLE 1. (continued)

14
$(\O/64)(3003/16 - 4165/16\ m + 1841/8\ m^2 - 532/4\ m^3 + 196/4\ m^4 - 21/2\ m^5 + m^6)$
$(\O/128)(4387/16 - 5562/16\ m + 2285/8\ m^2 - 310/2\ m^3 + 216/4\ m^4 - 22/2\ m^5 + m^6)$
$(\O/128)(2786/16 - 2794/16\ m + 444/4\ m^2 - 88/2\ m^3 + 20/2\ m^4 - m^5)$
$(\O/256)(7599/32 - 3525/16\ m + 1045/8\ m^2 - 195/4\ m^3 + 21/2\ m^4 - m^5)$
$(\O/256)(2063/16 - 731/8\ m + 157/4\ m^2 - 19/2\ m^3 + m^4)$
$(\O/512)(2655/16 - 870/8\ m + 175/4\ m^2 - 20/2\ m^3 + m^4)$
$(\O/512)(592/8 - 139/4\ m + 18/2\ m^2 - m^3)$
$(\O/1024)(714/8 - 156/4\ m + 19/2\ m^2 - m^3)$
$(\O/1024)(244/8 - 17/2\ m + m^2)$
$(\O/2048)(276/8 - 18/2\ m + m^2)$
$(\O/2048)(16/2 - m)$
$(\O/4096)(17/2 - m)$
$\O/4096$
$\O/8192$

TABLE 1. (continued)

15
$(\varnothing/128)(6435/16 - 9949/16\ m + 5066/8\ m^2 - 3525/8\ m^3 + 836/4\ m^4 - 130/2\ m^5 + 24/2\ m^6 - m^7)$
$(\varnothing/128)(4387/16 - 5562/16\ m + 2285/8\ m^2 - 310/2\ m^3 + 216/4\ m^4 - 22/2\ m^5 + m^6)$
$(\varnothing/256)(25147/64 - 14649/32\ m + 5615/16\ m^2 - 1485/8\ m^3 + 237/4\ m^4 - 23/2\ m^5 + m^6)$
$(\varnothing/256)(7599/32 - 3525/16\ m + 1045/8\ m^2 - 195/4\ m^3 + 21/2\ m^4 - m^5)$
$(\varnothing/512)(10254/32 - 4395/16\ m + 1220/8\ m^2 - 215/4\ m^3 + 22/2\ m^4 - m^5)$
$(\varnothing/512)(2655/16 - 873/8\ m + 175/4\ m^2 - 20/2\ m^3 + m^4)$
$(\varnothing/1024)(3369/16 - 513/4\ m + 97/2\ m^2 - 21/2\ m^3 + m^4)$
$(\varnothing/1024)(714/8 - 156/4\ m + 19/2\ m^2 - m^3)$
$(\varnothing/2048)(852/8 - 174/4\ m + 20/2\ m^2 - m^3)$
$(\varnothing/2048)(138/4 - 18/2\ m + m^2)$
$(\varnothing/4096)(155/4 - 19/2\ m + m^2)$
$(\varnothing/4096)(17/2 - m)$
$(\varnothing/8192)(18/2 - m)$
$\varnothing/8192$
$\varnothing/16384$

TABLE 1. (continued)

16
$(\emptyset/128)(6435/16 - 9949/16\ m + 5066/8\ m^2 - 3525/8\ m^3 + 836/4\ m^4 - 130/2\ m^5 + 24/2\ m^6 - m^7)$
$(\emptyset/256)(76627/128 - 43947/64\ m + 25879/32\ m^2 - 8485/16\ m^3 + 1090/8\ m^4 - 283/4\ m^5 + 23/2 m^6 - m$
$(\emptyset/512)(35401/64 - 9502/16\ m + 6835/16\ m^2 - 825/4\ m^3 + 259/4\ m^4 - 24/2\ m^5 + m^6)$
$(\emptyset/512)(10254/32 - 4395/16\ m + 1220/8\ m^2 - 215/4\ m^3 + 22/2\ m^4 - m^5)$
$(\emptyset/1024)(13623/32 - 5421/16\ m + 1414/8\ m^2 - 236/4\ m^3 + 23/8\ m^4 - m^5)$
$(\emptyset/1024)(3369/16 - 513/4\ m + 97/2\ m^2 - 21/3\ m^3 + m^4)$
$(\emptyset/2048)(4221/16 - 600/4\ m + 107/2\ m^2 - 22/2\ m^3 + m^4)$
$(\emptyset/2048)(852/8 - 174/4\ m + 20/2\ m^2 - m^3)$
$(\emptyset/4096)(1007/8 - 193/4\ m + 21/2\ m^2 - m^3)$
$(\emptyset/4096)(155/4 - 19/2\ m + m^2)$
$(\emptyset/8192)(173/4 - 20/2\ m + m^2)$
$(\emptyset/8192)(18/2 - m)$
$(\emptyset/16384)(19/2 - m)$
$\emptyset/16384$
$\emptyset/32768$

$$T(N,P) = T(N,N) = \varnothing/2^{N-1}$$

$$T(N,P) = T(N,N+2) = \varnothing/2^{N} \Big[(N + 5)/2 - m\Big]$$

$$T(N,P) = T(N,N+4) = \varnothing/2^{N+1} \Big[(N^2 + 13N + 46)/8 - (N + 8)m/2 + m^2)\Big]$$

$$\begin{aligned} T(N,P) = T(N,N+6) = \varnothing/2^{N+2} \Big[& (N^3 + 24N^2 + 203N + 612)/48 - (N^2 + 19N + 96)m/8 \\ & + (N + 11)m^2/2 - m^3) \end{aligned}$$

$$\begin{aligned} T(N,P) = T(N,N+8) = \varnothing/2^{N+3} \Big[& (N^4 + 38N^3 + 568N^2 + 3886N + 10632)/384 - (N^3 + 33N^2 + 380N + 1536)m/48 \\ & + (N^2 + 25N + 164)m^2/8 - (N + 14)m^3/2 + m^4)\Big] \end{aligned}$$

$$\begin{aligned} T(N,P) = T(N,N+10) = \varnothing/2^{N+4} \Big[& (N^5 + 55N^4 + 1245N^3 + 14585N^2 + 88994N + 227760)/3840 \\ & - (N^4 + 50N^3 + 971N^2 + 8722N + 30720)m/384 \\ & + (N^3 + 42N^2 + 611N + 3090)m^2/48 - (N^2 + 31N + 250)m^3/8 \\ & + (N + 17)m^4/2 - m^5)\Big] \end{aligned}$$

$$\begin{aligned} T(N,P) = T(N,N+12) = \varnothing/2^{N+5} \Big[& (N^6 + 75N^5 + 2395N^4 + 41865N^3 + 424564N^2 + 238060N + 5798880)/46080 \\ & - (N^5 + 70N^4 + 2015N^3 + 29930N^2 + 23030N + 737280)m/3840 \\ & + (N^4 + 62N^3 + 1487N^2 + 16402N + 70416)m^2/384 \\ & - (N^3 + 51N^2 + 896N + 5436)m^3/48 + (N^2 + 37N + 354)m^4/8 \\ & - (N + 20)m^5/2 + m^6)\Big] \end{aligned}$$

Figure 1. Gregory-Newton Transposition

TABLE 2. CONSTANT TERM NUMERATORS

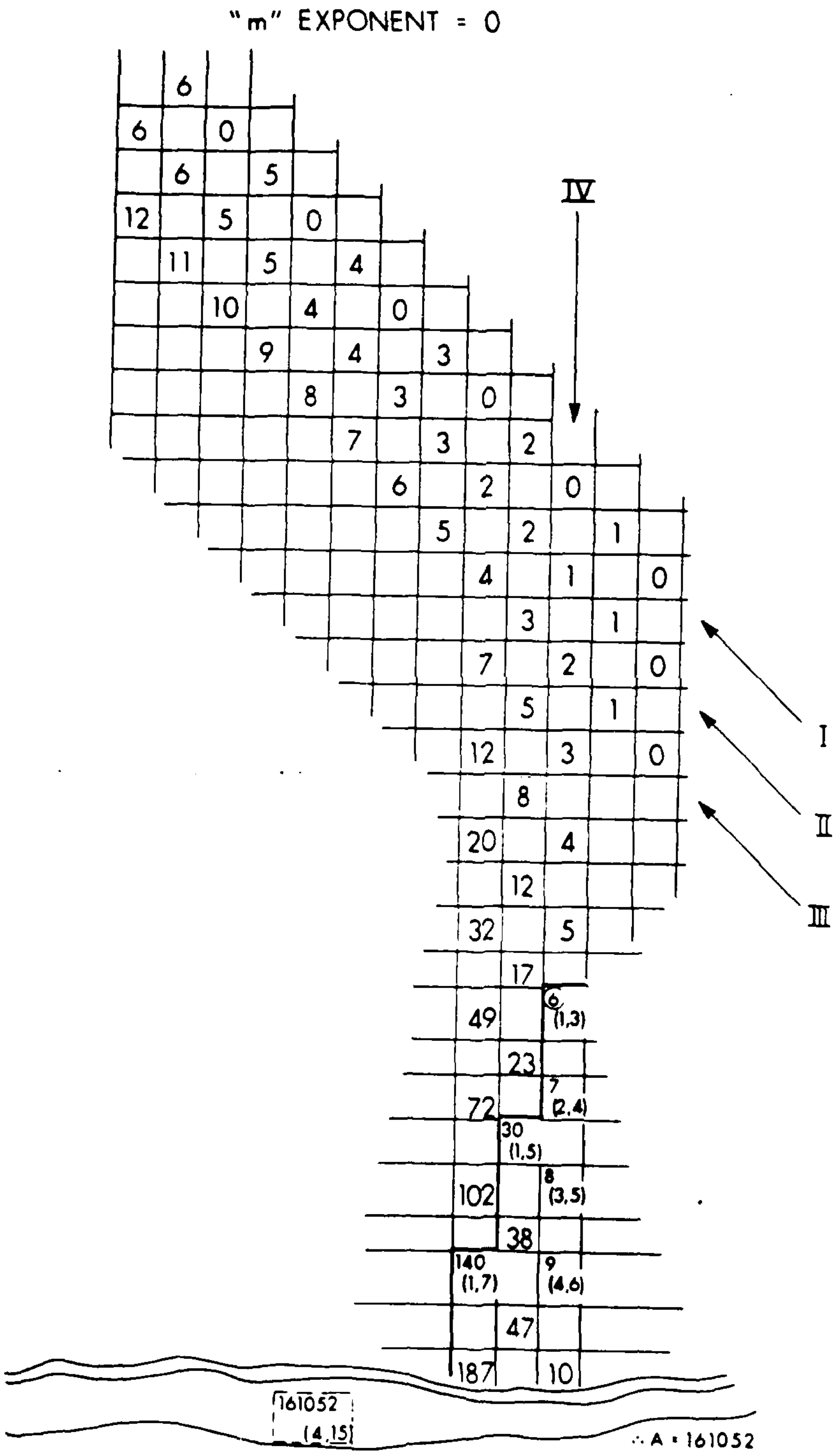


TABLE 3. POLYNOMIAL EXPANSIONS FOR SELECTED INTERSECTIONS

$$T(2,14) = 0/128 + 17548/64 - 11124m/32 + 4570m^2/16 - 1240m^3/8 + 216m^4/4 - 22m^5/2 + m^6$$

$$T(3,15) = 0/256 - 25147/64 - 14649m/32 + 5615m^2/16 - 1435m^3/8 + 237m^4/4 - 23m^5/2 + m^6$$

$$T(4,16) = 0/512 + 35401/64 - 19044m/32 + 6835m^2/16 - 1650m^3/8 + 259m^4/4 - 24m^5/2 + m^6$$

$$T(5,16) = 0/1024 + 49024/64 - 24465m/32 + 8249m^2/16 - 1886m^3/8 + 282m^4/4 - 25m^5/2 + m^6$$

$$T(6,18) = 0/2048 + 66868/64 - 31086m/32 + 9877m^2/16 - 3344m^3/8 + 306m^4/4 - 26m^5/2 + m^6$$

TABLE 4. 1ST POWER NUMERATORS

"m" EXPONENT = 1

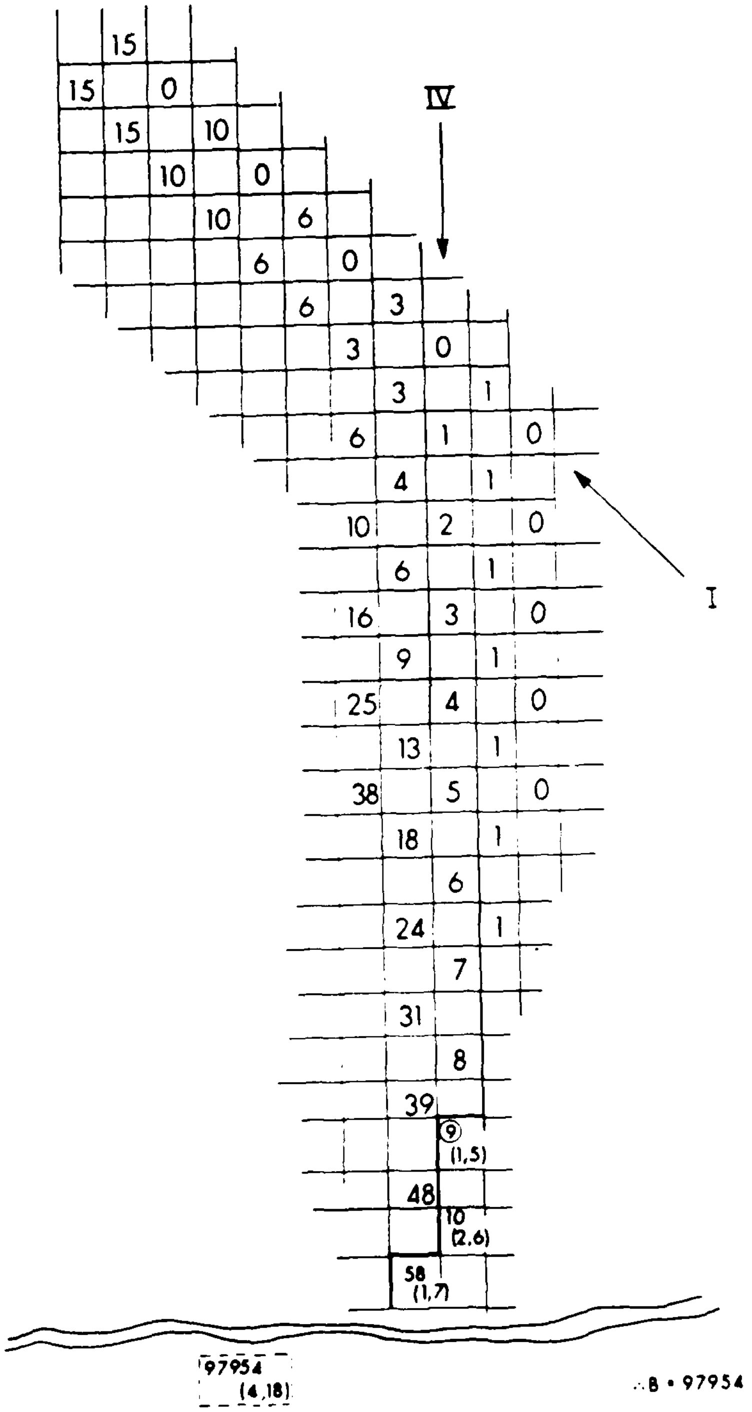
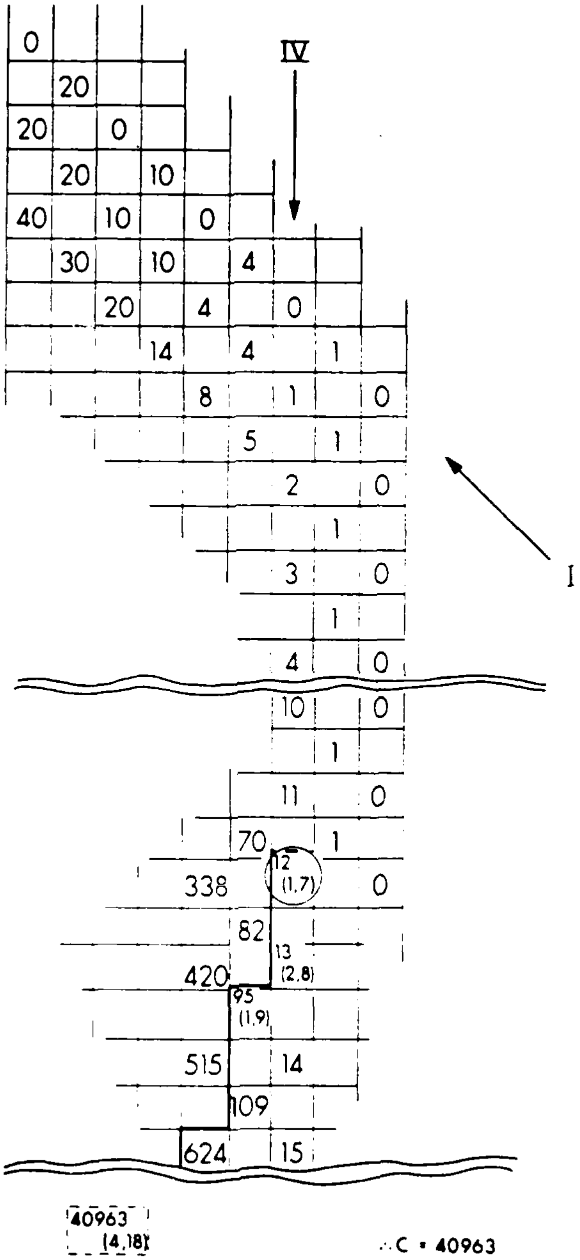


TABLE 5. 2ND POWER NUMERATORS

"m" EXPONENT = 2



∴C = 40963

TABLE 6. 3RD POWER NUMERATORS

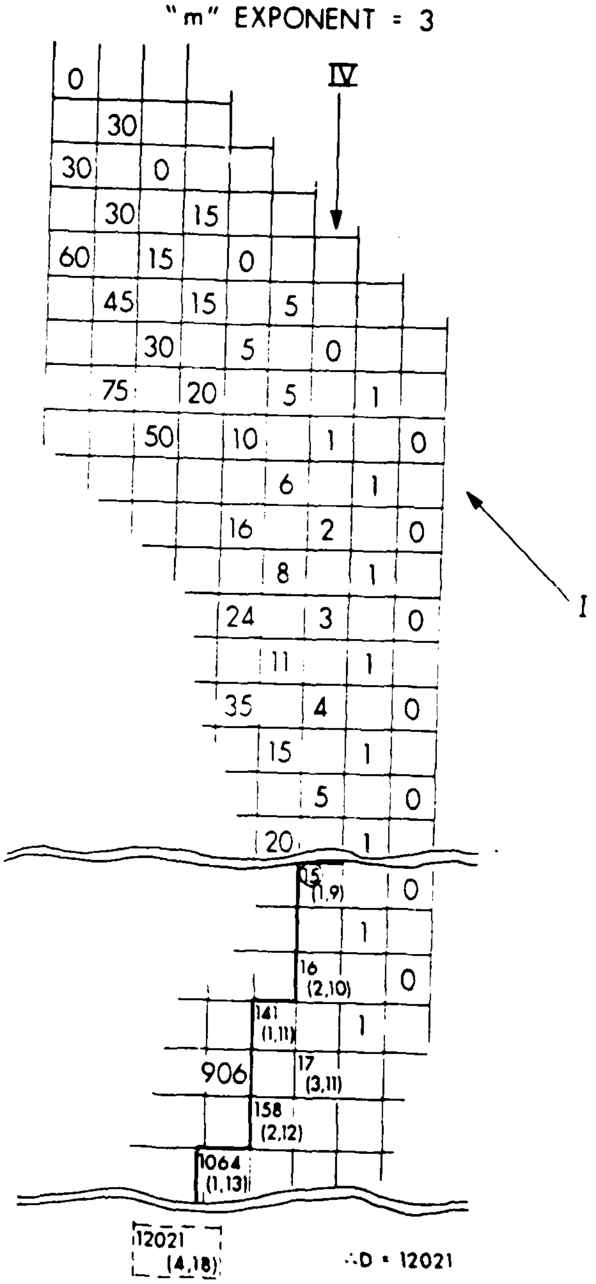


TABLE 7. 4th Power Numerators

"m" EXPONENT = 4

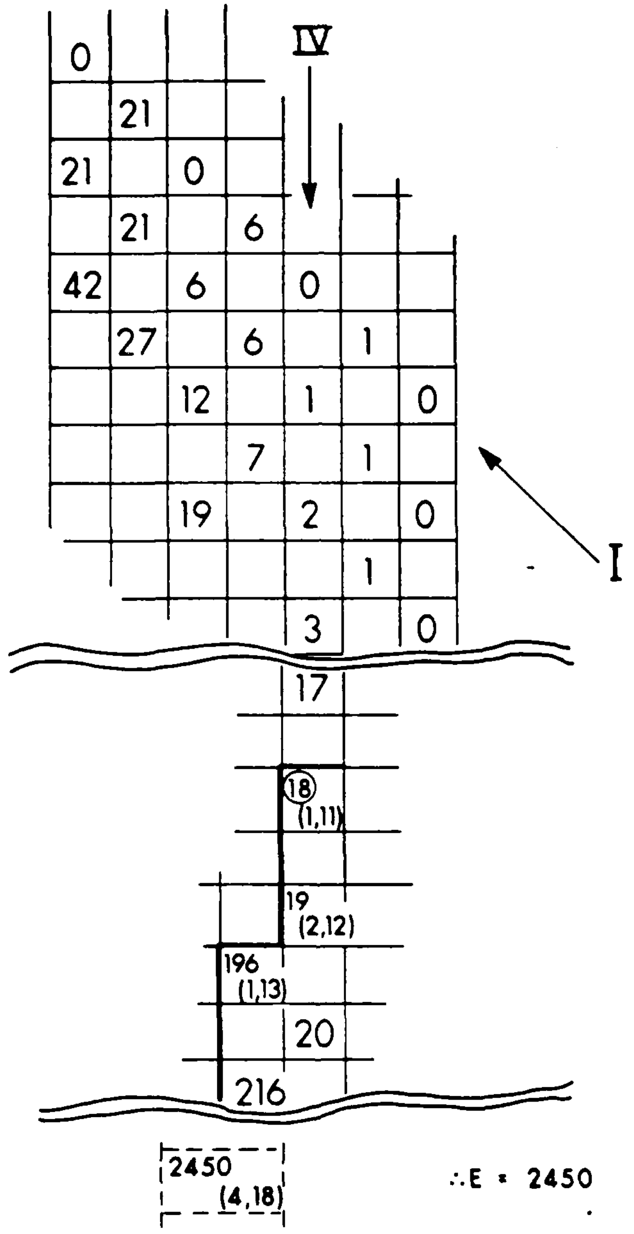


TABLE 8. 5TH POWER NUMERATORS

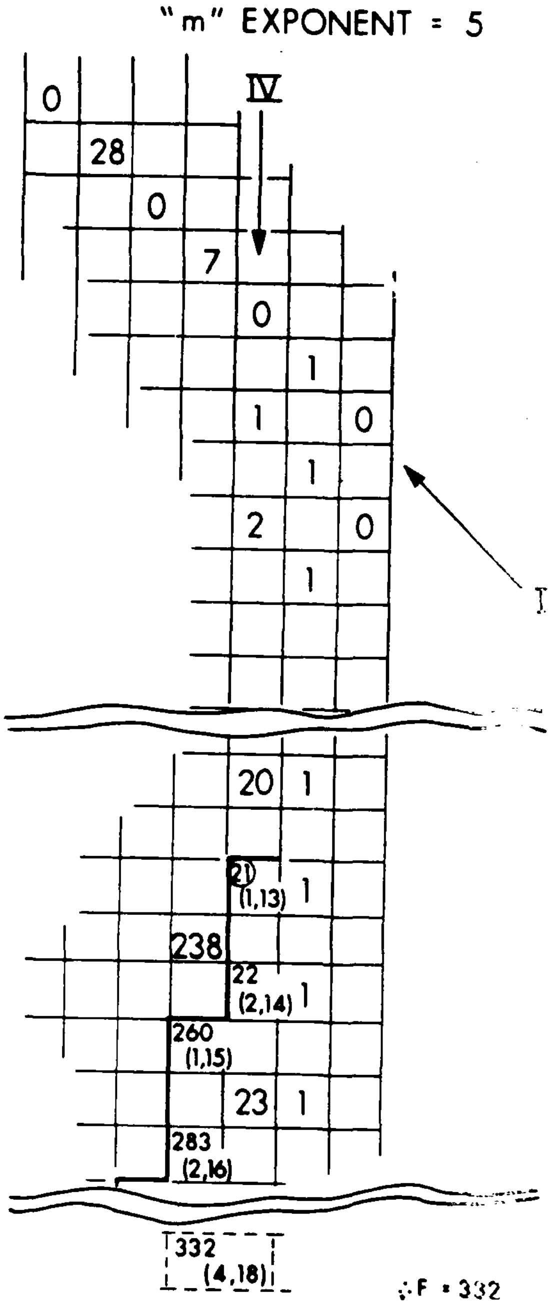
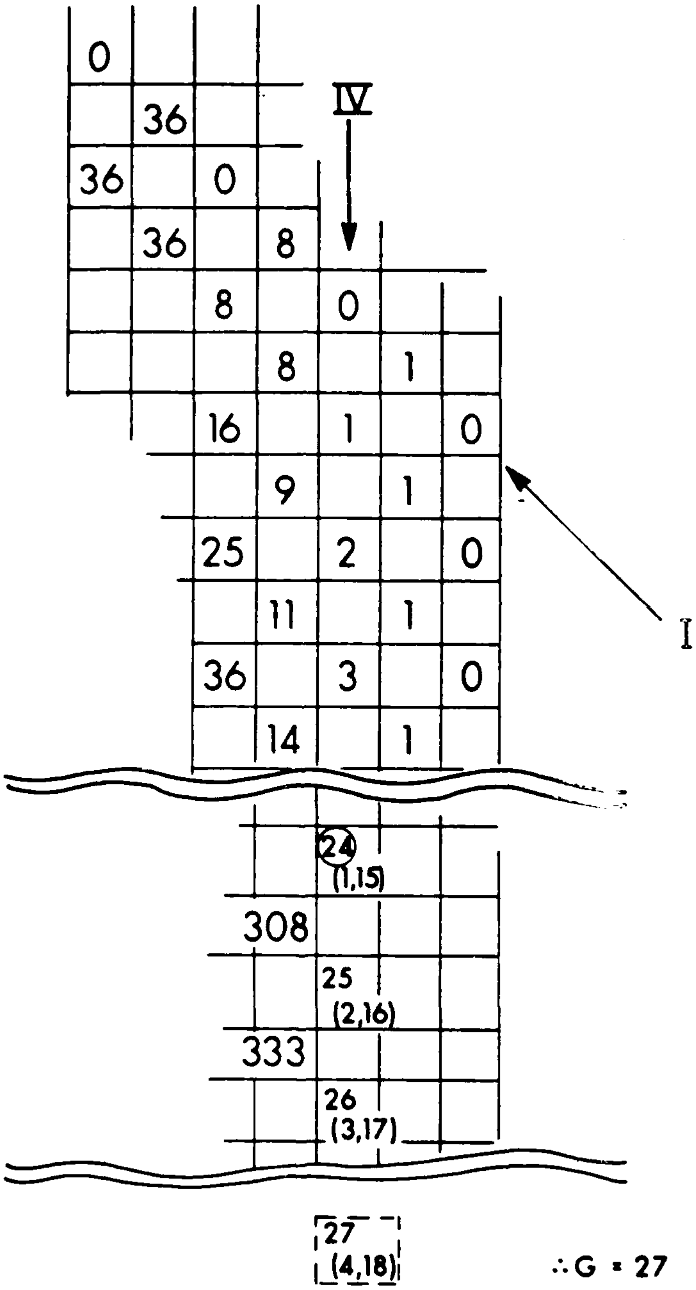


TABLE 9. 6TH POWER NUMERATORS

"m" EXPONENT = 6



∴ G = 27

TABLE 10. LINE PROGRESSION CORRELATIONS

Diagonal	Term Exponent	Column Value									
I	0	0	1	0	2	0	3	0	4	0	5
	1	0	1	0	3	0	6	0	10	0	15
	2	0	1	0	4	0	10	0	20		
	3	0	1	0	5	0	15				
	4	0	1	0	6	0	21				
	5	0	1	0	7						
II	0	0	1	1	2	2	3	3	4	4	5
	1	0	1	1	3	3	6	6	10	10	
	2	0	1	1	4	4	10	10	20	20	
	3	0	1	1	5	5	15	15			
	4	0	1	1	6	6	21	21			
	5	0	1	1	7						
	6	0	1	1	8						
III	0	0	1	2	3	4	5	6	7	8	9
	1	0	1	2	4	6	9	12	16	20	
	2	0	1	2	5	8	14	20	30		
	3	0	1	2	6	10	20	30			
	4	0	1	2	7	12	27				
	5	0	1	2	8						
	6	0	1	2	9						

TABLE 11. VERTICAL COUNTER CORRELATION

Term Exponent	"IV" Column Value	Distance-Time Coordinate
0	6	(1,3)
1	9	(1,5)
2	12	(1,7)
3	15	(1,9)
4	18	(1,11)
5	21	(1,13)
6	24	(1,15)
7	27	(1,17)
8	30	(1,19)
9	33	(1,21)

ACKNOWLEDGEMENT

The finite difference and difference equation path followed in this report must be ascribed to Mr. Stowell Elder, who also graciously expended time, effort and encouragement to complete the development.

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2. Spiegel, M.R., Theory and Problems of Finite Differences and Finite Difference Equations, Schaum's Outline Series in Mathematics, McGraw-Hill Book Company, New York, etc., 1971, pp. 36-44.

LIST OF SYMBOLS

a	constant (diffusivity)
h	exponent of 2 in the external denominator
j	exponent of 2 in each term denominator
k	exponent of "m" in final term
m	constant $\left(\frac{\Delta \chi}{\Delta \chi + \Delta \chi_o}\right)$
n	number of χ increments
t	independent variable (time)
χ	independent variable (distance)
A,B,C,...	numerical coefficients
N	distance index
P	time index
T	Dependent variable (temperature)
∂	indicating partial differentrative
Δ	indicating difference
$\Delta_t T$	time variable effecting a change in T
$\Delta_\chi T$	distance variable effecting a change in T
¢	external numerator

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